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FORMULA SHEET

CFA Level III: Private Markets

Chartered Financial Analyst · Specialised pathway + core

104 formulas **11** subject areas

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HOW TO USE THIS SHEET

This sheet covers the Private Markets pathway plus the four core areas every Level III candidate sits. The pathway sections come first; the shared core follows.

Private Markets is the most numerate of the three pathways. Multiples, waterfalls, leverage and cap rates all appear as calculations — and the calculations are unforgiving because a single misplaced fee changes every downstream number.

The recurring theme is that reported private-market returns are not what they appear. NAV is an appraisal, not a price. IRR is sensitive to timing and can be engineered. Multiples ignore time entirely. Understanding what each metric HIDES is worth more than being able to compute it.

The ▸ notes flag the judgement calls the essay session actually tests — where a metric misleads, and what you would say to an investment committee about it.

PRIVATE INVESTMENTS — PERFORMANCE METRICS · 2 formulas

The three multiples. They tell you different things, and a GP will quote whichever one flatters them.

Distributed to paid-in (DPI)

```
DPI = Cumulative distributions / Paid-in capital
```

Where: Distributions = realised cash actually returned to LPs; paid-in = capital called and funded.

▸ *The only multiple that cannot be flattered by a generous mark. DPI is real money in the LP's bank account — "the multiple you can spend".*

Total value to paid-in (TVPI)

```
TVPI = DPI + RVPI = (Distributions + NAV) / Paid-in capital
```

Where: NAV = the GP's own valuation of what is still held.

▸ *TVPI includes unrealised value that the GP marks themselves. Early in a fund's life it is almost entirely an opinion — treat a young fund's TVPI accordingly.*

> **EXAM TIPS — PRIVATE INVESTMENTS — PERFORMANCE METRICS**
>
> - DPI is cash, RVPI is opinion, TVPI is the sum. When you assess a GP's track record, ask what proportion of TVPI is DPI — the answer tells you whether they can actually exit.
> - None of the multiples account for time. A 2.0× over four years and a 2.0× over ten years are the same multiple and wildly different investments. Always pair a multiple with an IRR.

GP & LP ECONOMICS · 4 formulas

Fees, carry and the J-curve. This is where the LP's return actually gets decided.

Residual value to paid-in (RVPI)

```
RVPI = Residual NAV / Paid-in capital
TVPI = DPI + RVPI
```

Where: The unrealised portion of TVPI.

▸ *A declining RVPI over a fund's life is a GOOD sign — it means value is being realised and converted into DPI. A stubbornly high RVPI late in the fund life means exits are not happening.*

The J-curve

```
Early:  negative cash flows (calls + fees, no distributions) ⇒ negative net IRR
Inflection:  distributions begin to exceed contributions
```

Where: The shape of a fund's net IRR over its life.

▸ *Fees are charged from day one; value is realised years later. A negative IRR in years 1–3 is structural, not a verdict — judging a fund on its year-two IRR is the classic LP mistake.*

Management fee base

```
Investment period:  fee on COMMITTED capital
Post-investment period:  fee on INVESTED (deployed) capital
Fee = Rate × Base   (typically 1.5–2%)
```

Where: The base changes; the rate usually does not.

▸ *Fees on committed capital mean the LP pays on money that has not been put to work yet. That is a direct cause of the J-curve, and a legitimate point of negotiation.*

GP catch-up (100% catch-up provision)

```
X = [c / (1 − c)] × Pref
```

Where: c = carry rate (e.g. 0.20), Pref = the preferred return already distributed to LPs.

▸ *After the LPs get their hurdle, the GP takes 100% of distributions until they hold their full carry percentage of total profits. It is not extra money — it is the GP catching up to the agreed split.*

EXAM TIPS — GP & LP ECONOMICS

- Learn the waterfall as a sequence and write it out before computing anything: return of capital ⇒ preferred return (hurdle) ⇒ GP catch-up ⇒ carry split (typically 80/20).
- American (deal-by-deal) waterfalls pay the GP earlier and expose LPs to clawback risk. European (whole-fund) waterfalls make LPs whole first. The LP always prefers European — say so and say why.
- Net IRR is after fees and carry. Gross IRR is before. A GP quoting gross returns without qualification is telling you something about themselves.

PRIVATE EQUITY · 7 formulas

LBO mechanics and venture valuation. The most calculation-heavy section of the pathway.

Multiple on invested capital (MOIC)

```
MOIC = (Distributions + Residual NAV) / Invested capital
```

Where: Effectively the same construction as TVPI.

▸ *MOIC ignores time entirely. It is the honest headline number; the IRR is the one that flatters a fast exit. Always look at both.*

Levered equity return

```
r_e = r_u + (D/E)(r_u − r_d)
```

Where: r_u = unlevered asset return, r_d = cost of debt.

▸ *Leverage multiplies the spread between asset return and borrowing cost — in both directions. If r_u falls below r_d, leverage accelerates the loss. This is the entire risk of the LBO model in one line.*

LBO value creation decomposition

```
ΔEquity = ΔEBITDA × M_entry + EBITDA_exit × ΔM + ΔDebt
```

Where: M = EV/EBITDA multiple, ΔDebt = debt paid down over the hold.

▸ *The three levers: operational improvement, multiple expansion, and deleveraging. A sponsor whose return came entirely from multiple expansion did not create value — they got lucky on timing, and the exam wants you to say so.*

Sponsor IRR

```
IRR = (Exit equity / Entry equity)^(1/n) − 1
```

Where: Exit equity = exit EV minus remaining debt; entry equity = the sponsor's cheque.

▸ *Shortening the hold period raises IRR without raising MOIC. This is why GPs like early exits and why LPs should look past IRR to the multiple.*

Post-money valuation (VC method)

$$\text{Post-money} = \text{Exit value} / (1 + r)^n$$

Where: r = the VC's required IRR (often 40%+), n = years to exit.

‣ *The eye-watering discount rate is a proxy for failure risk across the portfolio — most investments return zero. It is not a claim that this company is that risky; it is portfolio arithmetic.*

VC ownership and pre-money valuation

```
Ownership % = Investment / Post-money
Pre-money = Post-money − Investment
```

Where: Investment = new capital injected.

‣ *Ownership is calculated on POST-money, always. Dividing by pre-money overstates the stake — and that is the distractor every single time.*

Waterfall distribution

```
Return of capital ⇒ Preferred return (hurdle) ⇒ GP catch-up ⇒ Carry split
```

Where: Typical: 20% carry above an 8% hurdle.

‣ *Work the waterfall strictly in order and write each tier down. Skipping a tier corrupts every number after it — this is a place where showing your steps genuinely protects your marks.*

EXAM TIPS — PRIVATE EQUITY

- For any LBO question, build the bridge explicitly: entry EV ⇒ entry equity ⇒ debt paydown and EBITDA growth over the hold ⇒ exit EV ⇒ exit equity. Then compute the return.
- The three value-creation levers are the standard essay: operations, multiple, leverage. Rank them by quality — operational improvement is real and repeatable; multiple expansion is market beta wearing a suit.

PRIVATE DEBT · 4 formulas

Credit metrics for direct lending. Leverage and coverage, computed the way a credit committee computes them.

Total debt to EBITDA

```
Leverage = Total debt / EBITDA
```

Where: Total debt = senior + subordinated principal.

‣ *Typically 5–7× on quality buyout credits. Watch for "adjusted" or "pro forma" EBITDA — add-backs for hypothetical synergies are how a 7× deal gets presented as a 5× deal.*

Interest coverage ratio

```
Interest coverage = EBITDA / Interest expense
```

Where: Interest expense = total cash interest.

‣ *Leverage says how much debt; coverage says whether it can be serviced. In a floating-rate loan, coverage deteriorates when rates rise even though leverage has not changed at all.*

Fixed charge coverage ratio

```
FCCR = (EBITDA − CapEx) / (Interest + Mandatory principal amortisation)
```

Where: The denominator is total debt service.

‣ *The most conservative of the coverage ratios — it deducts maintenance CapEx and includes principal repayment. A business cannot skip either, so this is the one a lender actually trusts.*

Loan-to-cost (LTC)

```
LTC = Loan amount / Total project cost
```

Where: Total project cost = acquisition price plus development and soft costs.

‣ *Used in development lending where there is no stabilised value yet. LTV uses appraised value; LTC uses money actually spent — and in a development, spent money is the more honest denominator.*

EXAM TIPS — PRIVATE DEBT

- Covenants are the point of private debt. Maintenance covenants are tested every quarter and give the lender an early seat at the table; incurrence covenants only bite on a specific action. "Covenant-lite" means the lender gave that seat up.
- Direct lending is compensated for illiquidity and complexity, not for taking more credit risk. If a question implies the premium is pure credit risk, that is the misconception it wants you to correct.

PRIVATE SPECIAL SITUATIONS · 1 formulas

Distressed and opportunistic credit. Small section, memorable formula.

Expected IRR on a distressed debt position

```
E[IRR] = (E[Recovery] / P)^(1/T) − 1
```

Where: E[Recovery] = probability-weighted recovery per unit of face value, P = purchase price, T = expected holding period in years.

▸ *The return is driven by the recovery estimate and by TIME — and in distressed investing, time is the variable nobody controls. A restructuring that drags an extra two years destroys the IRR even if the recovery estimate was right.*

EXAM TIPS — PRIVATE SPECIAL SITUATIONS

- Distressed strategies split into two: "loan-to-own" (buy the fulcrum security and convert to equity through the restructuring) and trading the recovery (buy cheap, sell into the workout). Identify which one the vignette describes before you answer.
- The fulcrum security is the tranche where the value breaks — the one that converts to equity in a restructuring. Finding it is the whole analytical exercise.

PRIVATE REAL ESTATE · 7 formulas

NOI, cap rates and leverage. Every real estate question in the pathway starts with NOI.

Net operating income

```
NOI = EGI − Operating expenses
```

Where: EGI = potential gross income − vacancy and credit losses.

▸ *Operating expenses EXCLUDE debt service, depreciation and income tax. Deducting the mortgage from NOI is the single most common error in the topic — NOI is a property-level, unlevered figure.*

Direct capitalisation value

```
V = NOI / r_cap
```

Where: Stabilised NOI, normalised for occupancy and expenses; cap rate from comparable sales.

▸ *Only valid for stabilised, income-producing property. A half-let building or one mid-refurbishment has no stabilised NOI, so direct capitalisation gives you nonsense — use DCF instead.*

Gordon link between cap rate and discount rate

```
r_cap ≈ r − g
```

Where: r = property discount rate (IRR target), g = expected NOI growth.

▸ *This one equation explains cap rates. A low cap rate can mean high expected growth OR a low required return — it does NOT automatically mean the property is expensive. That distinction is a favourite essay point.*

Discounted cash flow value

$$V_0 = \Sigma\ CF_t/(1 + r)^t + [NOI_{n+1} / r_{exit}] / (1 + r)^n$$

Where: r_exit = exit (terminal) cap rate.

▸ *The exit cap rate usually dominates the answer. Prudence says assume an exit cap slightly ABOVE the going-in cap — the building will be older when you sell it.*

Cap-rate sensitivity of value

$$\%\Delta V \approx -\Delta r\_cap / r\_cap$$

Where: Δr_cap in decimal form.

▸ *At a 5% cap rate, a 50 bp widening costs you about 10% of value — with no change whatsoever in the building or its rent roll. This is why real estate is a rates asset in disguise.*

Loan-to-value

$$LTV = \text{Loan amount} / \text{Property value}$$

Where: Typically 60–75% for commercial real estate.

▸ *Lenders covenant on LTV and DSCR together. When values fall, LTV breaches can trigger a default even though the tenant is still paying the rent on time.*

Levered equity cash flow

$$CF\_equity,t = NOI_t - Interest_t - CapEx_t$$

Where: The cash actually reaching the equity holder.

▸ *NOI is unlevered and CapEx sits below it. Levered equity cash flow is far more volatile than NOI — which is precisely why real estate equity blows up in a downturn while the buildings keep collecting rent.*

> **EXAM TIPS — PRIVATE REAL ESTATE**
>
> - Build NOI correctly and 80% of any real estate question is done. Potential gross income ⇒ less vacancy and credit loss ⇒ EGI ⇒ less operating expenses ⇒ NOI. No debt, no tax, no depreciation.
> - The four quadrants of real estate: public/private × equity/debt. A REIT is public equity real estate and it behaves like a stock in the short run and like a building in the long run.

INFRASTRUCTURE · 1 formulas

Regulated assets. One formula, and it explains the whole asset class.

Allowed revenue under the regulated asset base model

$$\text{Allowed revenue} = RAB \times r\_allowed + \text{Depreciation} + \text{Opex}$$

Where: RAB = regulated asset base, r_allowed = the regulator's permitted return.

▸ *The regulator effectively sets the return, which is why regulated infrastructure is bond-like — and why the key risk is REGULATORY, not commercial. A rate review that cuts r_allowed does more damage than any recession.*

> **EXAM TIPS — INFRASTRUCTURE**
>
> - Infrastructure sits on a spectrum: regulated/availability-based assets (bond-like, stable, low growth) at one end; demand-based assets like toll roads and airports (equity-like, GDP-sensitive) at the other. Place the asset before you answer.
> - Greenfield means construction risk and no cash flow for years. Brownfield means an operating asset with a cash flow record. The risk profiles are not comparable, and neither are the required returns.

CORE 1 — CAPITAL MARKET EXPECTATIONS & ASSET ALLOCATION · 23 formulas

Forecasting inputs, then turning them into a policy portfolio. Sat by every Level III candidate regardless of pathway.

Mean-variance optimal portfolio weights

$$w^* = (1/\lambda) \times \Sigma^{-1} (\mu - r\_f \cdot 1)$$

Where: λ = risk aversion, Σ = covariance matrix, μ = expected returns.

▸ *MVO is famously input-sensitive: small changes in μ produce wild swings in weights, which is exactly why Black-Litterman and resampling exist.*

Corner portfolio blending

$$w_A = [E(R_P) - E(R_B)] / [E(R_A) - E(R_B)]$$
$$w_B = 1 - w_A$$

Where: A and B = the two adjacent corner portfolios bracketing the target return.

▸ *Any blend of two adjacent corner portfolios sits on the efficient frontier. Blending non-adjacent ones does not.*

Black-Litterman expected returns

$$\text{Equilibrium:} \quad \Pi = \delta \cdot \Sigma \cdot w_{mkt}$$
$$\text{Blended:} \quad E(R) = [(\tau\Sigma)^{-1} + P^{T}\Omega^{-1}P]^{-1} \, [(\tau\Sigma)^{-1}\Pi + P^{T}\Omega^{-1}Q]$$

Where: δ = risk aversion, w_{mkt} = market-cap weights, Q = your views, Ω = confidence in them.

▸ *You will not compute the matrix algebra in the exam. You will be asked why it beats raw MVO: it anchors on market equilibrium and tilts only where you have a view.*

Rebalancing trigger (range-based)

$$\text{Rebalance when } |w_i - w^*_i| > \Delta_i$$

Where: w^*_i = target weight, Δ_i = tolerance band.

▸ *Wider bands mean lower transaction costs but more drift from policy. Bands widen with higher transaction costs and with higher correlation to the rest of the portfolio.*

Grinold-Kroner expected equity return

$$E(R) = D/P - \Delta S + g + \Delta(P/E)$$

Where: D/P = dividend yield, ΔS = net share issuance, g = nominal earnings growth, $\Delta(P/E)$ = repricing.

▸ *Note the sign on ΔS: net buybacks (negative issuance) add to return. Income + growth + repricing — that is the whole model in three words.*

Taylor rule policy rate

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - y^*)$$

Where: r^* = neutral real rate, π^* = inflation target, $(y - y^*)$ = output gap.

▸ *A rate above the Taylor-implied level suggests restrictive policy. It is a benchmark for judging central banks, not a prediction of them.*

Covered interest rate parity

$$F / S = (1 + i_d) / (1 + i_f)$$

Where: F, S quoted as domestic per foreign.

▸ *The high-yield currency always trades at a forward discount. If it did not, you could arbitrage it — which is why CIP holds tightly in practice.*

Fixed-income expected return building blocks

$$E(R) = r_f + \pi^e + TP + CP + LP$$

Where: TP = term premium, CP = credit premium, LP = liquidity premium.

▸ *Build the required return by stacking compensations for each risk you are actually bearing. Strip out the ones you are not.*

Expected fixed-income return decomposition

$$E(R_{FI}) = YTM + \text{Roll-down} + \Delta P_{yield} - L_{credit} - L_{FX}$$

Where: ΔP = price change from your yield-curve view, L = expected losses.

▸ *Roll-down is free return on an upward-sloping curve as the bond ages into a lower yield. It disappears on a flat curve.*

GARCH(1,1) variance forecast

$$\sigma^2_t = \omega + \alpha \cdot \varepsilon^2_{t-1} + \beta \cdot \sigma^2_{t-1}$$

Where: ω = long-run anchor, α = weight on the last shock, β = weight on the prior variance.

▸ *This is why volatility clusters. $\alpha + \beta$ close to 1 means shocks decay slowly and today's turbulence persists.*

Singer-Terhaar blended risk premium

$$RP = w(\sigma \cdot \rho \cdot SR_g) + (1 - w)(\sigma \cdot SR_g)$$

Where: w = degree of global market integration, ρ = correlation with the global portfolio.

▸ *A fully integrated market only earns compensation for its correlated (systematic) risk. A fully segmented one earns compensation for all its risk — hence the higher premium.*

TAA band around the strategic allocation

$$w_{TAA} \in [w_{SAA} - b,\ w_{SAA} + b]$$

Where: b = the tactical band set in the IPS (often ±5%).

▸ *Tactical deviations must live inside the IPS bands. A recommendation that breaches them is wrong no matter how good the market view is.*

Net after-cost tactical premium

$$\alpha_{net} = \alpha_{gross} - TC - T$$

Where: TC = transaction costs, T = realised tax cost.

▸ *Tactical calls are gross-alpha ideas that die on net-alpha arithmetic. Always net down before recommending the trade — this is a standard essay point.*

Stress liquidity coverage ratio

$$LCR_{stress} = \text{Accessible liquid assets (stress)} / \text{Cash demands (stress)}$$

Where: Both measured under a stressed scenario, not a normal one.

▸ *For illiquid-heavy portfolios, a prudent floor is around 2×. Capital calls arrive precisely when markets are down — the denominator and numerator move together, against you.*

Endowment spending rate

$$s = \text{Annual spending} / \text{Portfolio value}$$

Where: e.g. 5% of $800M = $40M.

▸ *The spending rate is the anchor for the required return. Raise spending and you must raise required return, which means raising risk.*

Correlation inputs required for MVO

$$N_\rho = n(n - 1) / 2$$

Where: n = number of asset classes.

▸ *Ten asset classes need 45 correlations. This combinatorial explosion is the practical argument against very granular MVO.*

Total MVO inputs required

$$N = 2n + n(n - 1)/2$$

Where: n expected returns + n standard deviations + n(n−1)/2 correlations.

▸ *Every extra input is an extra estimation error you feed into an optimiser that will happily maximise it.*

Pension funded ratio

$$FR = A / L$$

Where: A = market value of plan assets, L = PV of liabilities (e.g. PBO).

▸ *A well-funded plan should de-risk — it has already won. Under-funded plans face pressure to take more risk, which is exactly when they can least afford it.*

Pension surplus

$$S = A - L$$

Where: Surplus optimisation maximises the return on S, not on A.

▸ *Focusing on asset return alone ignores that the liability is also a moving, rate-sensitive position. Surplus is the real risk being managed.*

Dollar duration

$$DD = MV \times D$$

Where: MV = market value (or PV of liabilities), D = modified/effective duration.

▸ *The unit of account for LDI hedging. You match dollar duration of assets to dollar duration of liabilities — not durations, dollar durations.*

Mean-variance utility

$$U = E(R_p) - 0.5 \times \lambda \times \sigma^2_p$$

Where: λ = risk aversion (typically 1–10).

‣ *Higher λ means variance is penalised harder, so the optimal portfolio sits further left on the frontier. Set λ = 0 and you get the maximum-return corner.*

Geometric mean approximation

$$G \approx A - 0.5\sigma^2$$

Where: A = arithmetic mean, σ² = variance of returns.

‣ *The volatility drag. Two portfolios with the same arithmetic mean but different volatility do not compound the same — the volatile one ends up poorer.*

Roy's safety-first ratio

$$SF = [E(R_p) - R\_L] / \sigma_p$$

Where: R_L = minimum acceptable (threshold) return.

‣ *Maximise SF and, under normality, you minimise the probability of falling below R_L. Set R_L = R_f and it collapses into the Sharpe ratio.*

EXAM TIPS — CORE 1 — CAPITAL MARKET EXPECTATIONS & ASSET ALLOCATION

- Asset allocation essays are graded on justification, not arithmetic. State the constraint, state the number, state the conclusion — in that order.
- Whenever a question describes an investor, extract five things before you compute anything: return requirement, risk tolerance, time horizon, liquidity needs, and legal/tax/unique constraints.
- Volatility drag ($G \approx A - 0.5\sigma^2$) is the quiet reason risk control creates return. Expect it to be tested indirectly, through a comparison of two portfolios.

CORE 2 — RISK BUDGETING, TRADING & INVESTOR TYPES · 28 formulas

Decomposing portfolio risk, measuring execution cost, and the return requirements of each institutional and private investor type.

Marginal contribution to risk

$$MCTR_i = \beta_i \times \sigma_p$$
$$\beta_i = Cov(R_i, R_p) / \sigma^2_p$$

Where: Risk added by a marginal increase in asset i's weight.

‣ *Note the beta is to the portfolio, not to the market. This is the derivative of portfolio risk with respect to weight.*

Absolute contribution to risk

$$ACTR_i = w_i \times MCTR_i$$
$$\Sigma\ ACTR_i = \sigma_p$$

Where: The individual contributions sum exactly to total portfolio risk.

‣ *Risk budgeting means setting target ACTRs. An asset with a small weight can dominate the risk budget if its volatility and correlation are high.*

Tracking error (active risk)

$$TE = \sigma(R_p - R\_B)$$
$$TE\_annual = TE\_monthly \times \sqrt{12}$$

Where: Standard deviation of the active return series.

‣ *Scale by √12 from monthly, √252 from daily. Scaling the mean uses 12; scaling the standard deviation uses √12 — do not mix them.*

Implementation shortfall

```
IS = Explicit + Delay + Impact + Opportunity
```

Where: Explicit = commissions and fees; delay = decision-to-desk drift; impact = the price move you cause; opportunity = the return on shares you never filled.

▸ *This is the complete cost of a trade, including the cost of not trading. Trading urgently kills you on impact; trading slowly kills you on opportunity cost.*

Effective spread

```
Effective spread = 2 × |P_exec − P_mid|
```

Where: P_mid = quote midpoint at the time of execution.

▸ *The factor of 2 makes it round-trip comparable to the quoted spread. Effective spread below quoted spread means you got price improvement.*

Square-root market impact

```
Impact ∝ √(shares / ADV)
```

Where: ADV = average daily volume.

▸ *Doubling order size raises impact by about 41%, not 100%. This concavity is why breaking large orders into pieces helps — but only up to a point.*

VWAP transaction cost

```
Buy:   cost = P_exec − VWAP
Sell:  cost = VWAP − P_exec
```

Where: Positive means unfavourable in both cases.

▸ *VWAP flatters a patient trader and is easy to game. It is meaningless for an order that is itself a large share of the day's volume.*

Effective equity beta including private equity

```
β_eff = w_eq + w_PE × β_PE
```

Where: β_PE ≈ 1.3 (PE has levered equity beta).

▸ *Reported PE returns are smoothed by appraisal-based NAVs, which understates true beta. A portfolio that looks defensive may be levered equity in a slow-reporting wrapper.*

Effective PE allocation including unfunded commitments

```
w_PE(eff) = (NAV + UC) / (P + UC)
```

Where: UC = unfunded commitments, P = total portfolio value.

▸ *Ignoring unfunded commitments systematically understates PE exposure. The capital is committed — you just have not written the cheque yet.*

Total PE economic exposure

```
E_PE = NAV + UC
```

Where: What you are really on the hook for.

▸ *The denominator changes but the message does not: commitments are exposure. This is what caught endowments out in 2008.*

Liquidity coverage for an alternatives programme

$$LCR = L / CF_{12m}$$

Where: L = liquid assets, CF_{12m} = committed cash outflows over the next 12 months (capital calls + benefit payments).

▸ *The denominator includes both. An endowment paying a spending distribution and facing capital calls in the same drawdown is the classic liquidity failure.*

Endowment nominal return requirement

```
R_nom = s + c + π
```

Where: s = spending rate, c = management costs, π = expected inflation.

▸ *Additive approximation. Purists multiply: $(1 + s)(1 + c)(1 + \pi) - 1$. The exam accepts the additive form unless it says otherwise.*

Endowment real return requirement

$$R_{real} = s + c$$

Where: Inflation stripped out.

▸ *This is what "preserving intergenerational equity" actually requires: earn the spend plus the costs, in real terms, forever.*

Foundation minimum return

$$R \geq 0.05 + c + \pi$$

Where: 5% = the US IRS minimum annual distribution requirement.

▸ *The 5% is a legal floor, not a policy choice. It is the key structural difference between a private foundation and an endowment.*

Insurer duration-matched immunisation

$$D_A \times A = D_L \times L$$

Where: Match dollar duration of assets to dollar duration of liabilities.

▸ *Duration matching only immunises against small, parallel shifts. Convexity matching and periodic rebalancing are what make it hold.*

DV01 (price value of a basis point)

$$DV01 = D_{mod} \times MV \times 0.0001$$

Where: Dollar change in value per 1 bp yield move.

▸ *The hedging currency of LDI. Number of futures = ΔDV01 required ÷ DV01 per contract.*

Active share

$$AS = \tfrac{1}{2} \Sigma \, |w_{p,i} - w_{b,i}|$$

Where: Summed across the combined portfolio-plus-benchmark universe.

▸ *Active share and tracking error are different things. A well-diversified stock-picker can have high active share and low tracking error; a sector bet has the reverse.*

Required pre-tax return for a private client

$$r = [(S - I)/V + \pi] \,/\, (1 - t) + f$$

Where: S = spending, I = other income, V = portfolio value, t = tax rate, f = advisory fees.

▸ *Order of operations is the whole question: net the spending against income, express as a rate, add inflation, gross up for tax, then add fees. Grossing up in the wrong place is the standard error.*

Leveraged portfolio return on equity

$$r_p = r_i + (V_B / V_E)(r_i - r_B)$$

Where: V_B = borrowed value, V_E = equity, r_B = borrowing cost.

▸ *Leverage multiplies the spread between asset return and borrowing cost — in both directions. If $r_i < r_B$, leverage accelerates the loss.*

Human capital

$$HC = \Sigma \, E[w_t] \,/\, (1 + r)^t$$

Where: w_t = expected labour income, r = risk-adjusted discount rate.

▸ *A tenured professor has bond-like human capital and can hold more equities. A commission-based trader has equity-like human capital and should hold fewer. This is the entire life-cycle argument.*

Real after-tax return

$$r_{real,at} \approx r_{nom} - \pi - t \times r_{nom}$$

Where: Tax is levied on the nominal gain, including the inflation component.

▸ *Tax on inflation is a real loss of purchasing power. In high-inflation regimes a positive nominal after-tax return can still be a real loss.*

Modified duration from Macaulay

$$D_{mod} = D_{Mac} / (1 + y)$$

Where: y = periodic yield to maturity.

▸ *Carried straight over from Level I and still tested. Effective duration replaces it wherever cash flows are contingent.*

Taxable-equivalent yield

$$TEY = y_{muni} / (1 - t)$$

Where: t = investor's marginal tax rate.

▸ *The higher the client's bracket, the more attractive the muni. Compare TEY, never the raw muni yield, to a taxable bond.*

Economic net worth

$$ENW = HC + FC - PV(Liabilities)$$

Where: The holistic balance sheet.

▸ *Institutional thinking applied to an individual: assets include the PV of future earnings, liabilities include the PV of future consumption.*

SWF stabilisation fund sizing

$$AUM_{stab} = f \times G \times n$$

Where: f = fiscal dependence on the commodity, G = annual government spending, n = years of shortfall to cover.

▸ *A stabilisation fund is a short-horizon, low-volatility mandate — it may be called on next year. Do not let it hold illiquid growth assets.*

Norway-style fiscal transfer rule

$$T_{annual} = r_{real} \times V_{fund}$$

Where: $r_{real} \approx 3\%$ expected real return.

▸ *Spending only the expected real return preserves the principal in perpetuity. It is a savings fund, not a stabilisation fund — different horizon, different portfolio.*

Commodity SWF sector exposure cap

$$w_{energy} \le w_{bench} - \Delta$$

Where: Δ = a deliberate underweight (e.g. 30% of the benchmark weight).

▸ *The fund's inflows are already correlated with oil. Holding benchmark energy weight doubles down on the risk the country already carries — this is the concentration argument.*

SWF withdrawal cap

$$W_{max} = c \times (\text{3-year average AUM})$$

Where: c = charter cap (often 5%).

▸ *Averaging AUM over three years smooths withdrawals so a single bad market year does not force a spending collapse — the same logic as endowment smoothing.*

EXAM TIPS — CORE 2 — RISK BUDGETING, TRADING & INVESTOR TYPES

- Trading questions almost always resolve to the urgency trade-off: high urgency ⇒ market impact dominates; low urgency ⇒ opportunity cost and delay dominate. Match the algorithm to the motive for the trade.
- For any investor type, the exam wants: time horizon → liquidity need → risk tolerance → required return, in that causal order. Do not lead with the return number.
- Unfunded commitments are exposure. Any question featuring an endowment with a large PE programme is testing whether you know that.

CORE 3 — PERFORMANCE MEASUREMENT & ATTRIBUTION · 12 formulas

Risk-adjusted ratios and the Brinson decomposition. Reliably examined, and easy marks if you know which denominator to use.

Information ratio

$$IR = (\bar{R}_p - \bar{R}_B) / TE = \bar{\alpha} / TE$$

Where: $\bar{\alpha}$ = mean active return, TE = tracking error.

▸ *Active return per unit of active risk. Unlike Sharpe, IR is unaffected by adding cash or leverage to an unconstrained portfolio.*

Fundamental law of active management

$$IR = IC \times \sqrt{BR}$$
$$E(R_A) = IC \times \sqrt{BR} \times \sigma_A$$

Where: IC = information coefficient (skill), BR = breadth (independent bets per year).

▸ *Skill matters more than breadth because breadth is under a square root. Note that breadth must be independent — 500 correlated bets are not 500 bets.*

Sharpe ratio

$$SR_p = (R_p - R_f) / \sigma_p$$
$$SR^2_{combined} = SR^2_B + IR^2$$

Where: Total risk in the denominator.

▸ *The second identity is the beautiful one: adding an active overlay to a benchmark improves the combined Sharpe by exactly the manager's IR, squared.*

M-squared

$$M^2 = (R_p - R_f) \times (\sigma_m / \sigma_p) + R_f$$

Where: De-levers or levers the portfolio to the market's volatility.

▸ *Same ranking as Sharpe, but expressed in percent — so it can be compared directly against the market return.*

Treynor ratio

$$T = (R_p - R_f) / \beta_p$$

Where: Systematic risk in the denominator.

▸ *Correct when the portfolio is one sleeve of a diversified whole. Sharpe is correct when it is the client's entire wealth.*

Sortino ratio

$$Sortino = (R_p - MAR) / \sigma_d$$

Where: MAR = minimum acceptable return, σ_d = downside deviation below MAR.

▸ *Only penalises downside volatility. Preferred for strategies with deliberately asymmetric payoffs — option-selling, hedge funds, anything with a fat left tail.*

Brinson allocation effect

$$A_i = (w_{p,i} - w_{b,i})(R_{b,i} - R_b)$$

Where: Was the sector overweighted, and did that sector beat the total benchmark?

▸ *Allocation uses BENCHMARK sector returns. Being overweight a sector that outperformed the overall benchmark is a positive allocation effect — regardless of what you actually picked inside it.*

Brinson selection effect

$$S_i = w_{b,i} \times (R_{p,i} - R_{b,i})$$

Where: Did your picks beat the sector benchmark?

▸ *Selection uses BENCHMARK weights. Allocation ⇒ weight difference × benchmark return. Selection ⇒ benchmark weight × return difference. Getting these backwards is the most common attribution error there is.*

Upside capture ratio

$$UC = \bar{R}_{p,up} / \bar{R}_{b,up}$$

Where: Averaged over periods when the benchmark rose.

▸ *Above 100% means the manager amplifies up markets.*

Downside capture ratio

$$DC = \bar{R}_{p,down} / \bar{R}_{b,down}$$

Where: Averaged over periods when the benchmark fell.

▸ *Below 100% means the manager dampens losses — which is the more valuable of the two, because of compounding.*

Up/down capture ratio

$$Up/Down = Upside\ capture / Downside\ capture$$

Where: A single asymmetry number.

▸ *Above 1.0 is favourable convexity. A manager capturing 90% of the upside and 60% of the downside (ratio 1.5) beats one capturing 110% and 105%.*

Symmetric performance fee

$$\text{Fee} = \text{Base} + s \times (R_p - R_B)$$

Where: s = sharing rate.

▸ *Symmetric means the fee falls when the manager underperforms — genuine alignment. Asymmetric (upside-only) fees give the manager a free option on volatility, and the exam wants you to say so.*

EXAM TIPS — CORE 3 — PERFORMANCE MEASUREMENT & ATTRIBUTION

- Attribution: allocation is about WEIGHTS, selection is about PICKS. If you remember only one thing from this section, remember which return goes with which weight.
- Choosing the ratio is the question. Total risk ⇒ Sharpe or M^2. Systematic risk ⇒ Treynor or Jensen. Active risk ⇒ information ratio. Downside risk ⇒ Sortino.
- A high information ratio from a very low tracking error is fragile — it may just be a closet indexer with a lucky quarter. Look at active share alongside it.

CORE 4 — DERIVATIVES & CURRENCY RISK MANAGEMENT · 15 formulas

Hedging, overlays and option structures. The futures-contract formulas are near-guaranteed calculation marks.

Option delta

$$\text{Call: } \Delta_c = N(d_1) \in (0, 1)$$
$$\text{Put: } \Delta_p = N(d_1) - 1 \in (-1, 0)$$
$$\Delta_c - \Delta_p = 1$$

Where: Approximate change in option value per $1 move in the underlying.

▸ *Delta is also a rough probability of finishing in the money. ATM options sit near |0.5| and their delta moves fastest — that is gamma.*

Protective put

$$\text{Payoff} = S_T + \max(X - S_T, 0) = \max(S_T, X)$$
$$\text{Profit} = \text{Payoff} - (S_0 + p)$$

Where: Long stock + long put.

▸ *Insurance: floor the downside, keep the upside, pay a premium for the privilege. The put premium is the drag on a flat market.*

Covered call

$$\text{Payoff} = S_T - \max(S_T - X, 0) = \min(S_T, X)$$
$$\text{Profit} = \text{Payoff} - S_0 + c$$

Where: Long stock + short call.

▸ *Income in flat or mildly down markets, capped upside. You are selling the right tail to fund the present — fine until the stock gaps up.*

Collar

$$\text{Payoff} = S_T + \max(X_L - S_T, 0) - \max(S_T - X_H, 0)$$
$$= \min(\max(S_T, X_L), X_H)$$

Where: Long stock + long put (X_L) + short call (X_H).

▸ *A zero-cost collar funds the protective put by selling the call. Standard answer for a concentrated stock position the client cannot or will not sell.*

Put-call parity

$$S + P = C + PV(K)$$

Where: Same strike, same expiry, European.

▸ *Every synthetic in the Level III toolkit falls out of rearranging this. Long stock + long put = long call + cash.*

Long straddle breakevens

$$BE = K \pm (C_0 + P_0)$$

Where: Long call + long put at the same strike.

▸ *A pure volatility bet — you need a big move in either direction. You lose if the underlying sits still, and time decay is working against you the whole way.*

Bull call spread maximum gain

$$\text{Max gain} = (K_H - K_L) - \text{Net premium}$$

Where: Buy the lower strike, sell the higher.

▸ *Max loss is simply the net premium paid. Capping the upside is what makes the position cheap.*

Maximum loss on short stock + long call

$$\text{Max loss} = K - S_0 + C_0$$

Where: A synthetic long put.

▸ *The call caps the unlimited loss on the short. Recognising the synthetic is usually faster than working the payoff from scratch.*

Number of bond futures to adjust duration

$$N = (DD_T - DD_P) / DD_f$$

Where: DD = dollar duration; DD_f is BPV-adjusted by the conversion factor.

▸ *Positive N means buy futures to extend duration; negative means sell to shorten. The conversion factor for the cheapest-to-deliver bond is not optional.*

Number of equity futures to adjust beta

$$N = [(\beta_T - \beta_P) / \beta_F] \times [V / (P_f \times m)]$$

Where: β_T = target beta, m = contract multiplier.

▸ *Set $\beta_T = 0$ to fully hedge equity exposure; set $\beta_T = 1$ to equitise a cash pile. Same formula, both directions.*

Roll yield on a currency forward hedge

$$\text{Roll yield} = (F - S) / S$$

Where: Approximately the domestic minus foreign interest rate differential.

▸ *Hedging a high-yielding foreign currency costs you money (negative roll). This is why the hedging decision is never free, and why the exam asks about it.*

Minimum-variance hedge ratio

$$h^* = \rho_{A,B} \times (\sigma_A / \sigma_B)$$

Where: A = the asset being hedged, B = the hedging instrument.

▸ *It is just the slope coefficient from regressing A on B. If $\rho = 1$ and volatilities match, $h^* = 1$ and it is a perfect hedge — real cross-hedges never are.*

Domestic-currency return on a foreign asset

$$R_{DC} = (1 + R_{FC})(1 + R_{FX}) - 1$$

Where: R_{FX} = % change in the foreign currency against the domestic.

▸ *It is multiplicative, not additive. The cross term matters when both moves are large — and the exam picks large numbers deliberately.*

Variance notional from vega notional

$$N_{var} = N_{vega} / (2 \times \sigma_{strike})$$

Where: σ_{strike} in whole-number percent (e.g. 20, not 0.20).

▸ *Vega notional is intuitive (dollars per vol point); variance notional is what the contract actually pays on. Convert before you compute the payoff.*

Variance swap payoff

$$\text{Payoff} = N_{var} \times (\sigma^2_{realised} - \sigma^2_{strike})$$

Where: Volatilities squared — it settles on variance, not volatility.

▸ *Because it pays on variance, the payoff is convex in volatility: the long side gains more from a vol spike than it loses from an equivalent vol fall.*

EXAM TIPS — CORE 4 — DERIVATIVES & CURRENCY RISK MANAGEMENT

- Futures overlay questions are formulaic marks. Write down current exposure, target exposure, and the exposure per contract — then divide. The only real difficulty is a sign error.
- Currency hedging: the roll yield tells you the cost, the correlation tells you the benefit. A currency that already hedges the underlying asset (negative correlation) may be best left unhedged.
- When a question describes a concentrated low-basis stock position, it wants a collar, an exchange fund, or a completion portfolio — and it wants you to discuss the tax consequence of each.

LEVEL III EXAM STRATEGY

The constructed-response session

- Show the formula, then the substitution, then the answer. Waterfall questions in particular reward visible steps — a wrong number with correct working still scores.
- Answer the command word. "Calculate" wants the number. "Justify" wants the reason. "Evaluate" wants a judgement, supported.
- Allocate time by the printed minutes. A private-markets calculation can absorb an hour if you let it, and the marks are not there.

Private markets ideas that carry the most weight

- What each metric hides: IRR hides timing manipulation, multiples hide time, NAV hides the fact that it is an appraisal and not a price.
- The waterfall, in strict order, and the difference between American and European structures.
- Cap rate = r − g. This single relationship underlies every real estate valuation question.
- Unfunded commitments are real exposure, and capital calls arrive precisely when markets fall. Liquidity planning is not optional.
- The J-curve is structural. Do not judge a young fund on its net IRR.

A note on the pathways

- The core (labelled Core 1–4 here) is common to all three pathways and accounts for the larger share of the exam.
- Candidates fail Level III on asset allocation, performance evaluation and ethics far more often than on their chosen specialism. Do not let the pathway eat your revision time.