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FORMULA SHEET

CFA Level III: Portfolio Management

Chartered Financial Analyst · Specialised pathway + core

120 formulas **11** subject areas

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HOW TO USE THIS SHEET

This sheet covers the Portfolio Management pathway plus the four core areas every Level III candidate sits. The pathway sections come first; the shared core follows.

Level III is half constructed response. A formula you can only recognise is worth nothing — you need to be able to produce it, apply it, and then justify the answer in two sentences. Practise writing, not just calculating.

The ▸ notes are written for the essay paper: they flag the sentence the grader is actually looking for, not just the arithmetic.

Rule for the morning session: show the formula, show the substitution, box the answer, then write the one-line justification. Partial credit is real and it is generous — an unfinished, visible method beats a bare wrong number every time.

INDEX-BASED EQUITY STRATEGIES · 5 formulas

Why a tracker does not perfectly track. Every term here is a source of divergence from the index.

Tracking error decomposition

$$TE^2 = TE^2_{factor} + TE^2_{specific}$$

Where: Factor TE from active factor exposures; specific TE from stock-level bets.

▸ *Index funds try to drive both toward zero. A quant fund deliberately runs factor TE while suppressing specific TE — that difference is the exam point.*

Tracking error of an index portfolio

$$TE = \sigma(R_p - R_b)$$

Where: Standard deviation of the return differential, annualised.

▸ *Full replication ⇒ near-zero TE but high transaction cost in illiquid names. Sampling ⇒ cheaper but introduces sampling residual. That trade-off is the whole design decision.*

Index portfolio return decomposition

$$R_p = R_{idx} - f - c_t - \varepsilon_s + r_{lend} + r_{cash}$$

Where: f = fees, c_t = transaction costs, ε_s = sampling residual, r_{lend} = securities lending revenue, r_{cash} = cash spread.

▸ *Securities lending revenue is why some trackers beat their index gross of fees. It also introduces counterparty risk — say so if asked to evaluate it.*

Cash drag

$$Drag = w_{cash} \times (R_{eq} - R_{mm})$$

Where: w_{cash} = cash weight, R_{mm} = money-market rate earned on it.

▸ *In a rising market, uninvested cash costs you the equity risk premium on that slice. Equitising cash with futures removes the drag without giving up liquidity.*

Implicit financing cost of an equity index future

$$c_{carry} = r_f - d$$

Where: r_f = risk-free rate, d = index dividend yield.

▸ *This is embedded in the futures price. If the future is rich relative to fair value, the synthetic exposure is expensive — check before you equitise.*

EXAM TIPS — INDEX-BASED EQUITY STRATEGIES

- Full replication, stratified sampling and optimisation sit on one axis: tracking precision versus cost. Choose based on the index's breadth and the liquidity of its tail.
- Synthetic index exposure (futures, swaps) versus physical is a recurring essay: futures are cheap and liquid but need rolling; swaps give exact exposure but carry counterparty risk.

ACTIVE EQUITY INVESTING · 8 formulas

Measuring active bets, their capacity, and whether the manager has actual skill.

Jensen's alpha

$$\alpha = R_p - [R_f + \beta_p(R_m - R_f)]$$
$$\text{Fundamental law link: } E(\alpha) = IC \times \sqrt{BR} \times \sigma_A \times TC$$

Where: CAPM-adjusted outperformance.

▸ *The second identity is the more useful one at Level III: it says alpha comes from skill, breadth, active risk taken, and how efficiently you can implement.*

Fundamental law of active management

$$IR = IC \times \sqrt{BR} \times TC$$

Where: TC = transfer coefficient — how much of your view survives constraints.

▸ *The transfer coefficient is the Level III addition. Long-only constraints, position limits and turnover caps all push TC below 1, so a skilled manager can still deliver mediocre IR.*

Correlation-adjusted effective breadth

$$BR_{eff} = N / [1 + (N - 1)\rho]$$

Where: ρ = average pairwise correlation across signals.

▸ *Two hundred correlated bets on the same macro view are one bet. This formula is the mathematical version of "your portfolio is less diversified than you think".*

Active risk (tracking error)

$$\sigma_A = \sqrt{Var(R_P - R_B)}$$

Where: Standard deviation of active return.

▸ *Active risk budget is the scarce resource. The manager's job is to spend it where IC is highest — not to spread it evenly.*

Active share

$$\text{Active Share} = \tfrac{1}{2} \sum |w_{P,i} - w_{B,i}|$$

Where: Ranges from 0 (index) to 1 (no overlap).

▸ *Active share measures how DIFFERENT the holdings are; tracking error measures how differently they BEHAVE. A diversified stock-picker: high active share, low TE. A sector bet: the reverse.*

Expected active return from IR

$$R_A = IR \times TE$$

Where: Alpha earned = skill ratio × risk taken.

▸ *Rearranged, it tells you how much active risk you must run to hit a return target. If that number breaches the IPS band, the target is not feasible — a common essay conclusion.*

Treynor ratio

$$T_p = (R_p - R_f) / \beta_p$$

Where: Excess return per unit of systematic risk.

▸ *Use it when the sleeve is one component of a diversified whole — that is precisely the Level III context.*

Days-to-trade (capacity)

$$\text{Days-to-trade} = \text{Position } \$ / (0.10 \times \text{ADV } \$)$$

Where: Assumes you take no more than 10% of average daily volume per day.

▸ *This is the capacity constraint that kills small-cap strategies as AUM grows. If a position takes 20 days to exit, it is not a liquid position — regardless of what the fact sheet says.*

EXAM TIPS — ACTIVE EQUITY INVESTING

- Style questions resolve on the process: fundamental (bottom-up, concentrated, low breadth, high IC) versus quantitative (systematic, high breadth, low IC). The fundamental law explains why both can work.
- If asked to critique a manager, look at active share and tracking error TOGETHER. High fee plus low active share plus low TE is a closet indexer, and you should say so plainly.

LIABILITY-DRIVEN INVESTING · 4 formulas

Matching assets to liabilities. Dollar duration is the unit of account.

DV01

$$DV01 = (\text{Modified duration} \times V) / 10{,}000$$

Where: V = portfolio market value.

▸ *Dollar price change for a 1 bp move. Everything in LDI hedging is denominated in DV01, not in duration.*

Futures contracts for duration adjustment

$$N_f = [(D_{target} - D_{portfolio}) \times V_{portfolio}] / (D_{futures} \times V_{futures})$$

Where: Positive N_f ⇒ buy futures (extend duration); negative ⇒ sell (shorten).

▸ *Futures are the cheap, liquid way to change duration without disturbing the underlying holdings. Show the sign explicitly in the essay — graders look for it.*

Liability-driven surplus duration

$$\Delta Surplus = (D_A \times A - D_L \times L) \times \Delta y$$
$$\text{Immunised when: } D_A \times A = D_L \times L$$

Where: Surplus is immunised, not the assets.

▸ *A fully funded plan holding short-duration assets is NOT safe — when rates fall, liabilities rise faster than assets and the surplus evaporates. This is the single most important idea in LDI.*

Macaulay duration of a bond portfolio

$$D_{Mac} = \Sigma[t \times CF_t/(1 + y)^t] / \Sigma[CF_t/(1 + y)^t]$$

Where: Weighted average time to cash flow receipt.

▸ *For a single-liability immunisation, match Macaulay duration to the investment horizon, minimise convexity dispersion, and rebalance as time passes and yields move.*

EXAM TIPS — LIABILITY-DRIVEN INVESTING

- The three immunisation conditions for a single liability: PV of assets ≥ PV of liability, asset Macaulay duration = liability horizon, and minimise the dispersion of asset cash flows around that horizon.
- Cash flow matching eliminates interest-rate risk entirely but is expensive and inflexible. Immunisation is cheaper but requires rebalancing and only protects against small parallel shifts. Know when to recommend each.

YIELD CURVE STRATEGIES · 7 formulas

Positioning for a curve view. Every strategy here is a bet on level, slope or curvature.

Key rate duration

$$\%\Delta P \approx -\Sigma\, KRD_k \times \Delta y_k$$

Where: KRD_k = sensitivity to a shift at maturity point k.

▸ *The sum of key rate durations is approximately effective duration. KRDs are what let you express a steepener or a flattener — effective duration alone cannot.*

Expected bond return from a yield change

$$E(R) \approx Y + \text{Roll-down} - D_{mod} \cdot \Delta y + \tfrac{1}{2}C(\Delta y)^2$$

Where: Y = yield income.

▸ *The complete return picture: carry, roll, duration effect, convexity effect. Candidates who only compute the duration term consistently under-answer this.*

Expected return of a yield curve strategy

$$E(R) = Y + \text{Roll-down} + E(\Delta P_{view}) - E(\text{Credit loss}) \pm E(FX)$$

Where: Full decomposition including currency.

▸ *When a strategy relies on the curve NOT moving, most of the return is carry and roll-down. State that explicitly — it tells the grader you understand where the money comes from.*

Expected price change: your view vs the forwards

$$E(\Delta P) \approx -D(\Delta y_{view} - \Delta y_{fwd}) + \tfrac{1}{2}C(\Delta y_{view})^2$$

Where: Δy_{fwd} = the yield change already implied by forward rates.

▸ *This is the most important idea in the whole topic. You only profit if your view differs from what the forwards already price. Being right about rates rising earns you nothing if the market expected it.*

Barbell vs bullet convexity

$$\text{At equal duration: } C_{barbell} > C_{bullet}$$

Where: Barbell = short + long; bullet = concentrated in the middle.

▸ *Barbell wins when volatility is high or the curve moves non-parallel. Bullet wins in a stable, upward-sloping curve where you are paid for carry, not convexity.*

Butterfly spread

$$\text{Long butterfly profit condition: } 2y_{5Y} < y_{2Y} + y_{10Y}$$

Where: Long the belly, short the wings (or the reverse).

▸ *A pure curvature bet — duration-neutral, so a parallel shift produces nothing. If a question says "duration neutral but expects the curve to hump", it means a butterfly.*

Hedged cross-currency bond return (CIP)

$$R_{hedged} \approx R_{local} - (r_{local} - r_{base})$$

Where: Hedging strips out the interest-rate differential.

▸ *This is why hedged yield pickups mostly disappear. If a foreign bond yields more purely because its short rate is higher, hedging hands the pickup straight back to the market.*

EXAM TIPS — YIELD CURVE STRATEGIES

- Always ask what the forwards already imply before recommending a trade. "The curve will steepen" is not a view — "the curve will steepen MORE than the forwards imply" is.
- Match the structure to the view: level ⇒ duration. Slope ⇒ steepener/flattener via KRDs. Curvature ⇒ butterfly. Volatility ⇒ convexity (barbell, or long options).

CREDIT STRATEGIES · 6 formulas

Spread risk, excess return and the instruments used to express a credit view.

Credit spread duration

$$\%\Delta P \approx -\text{Spread duration} \times \Delta s$$

Where: Δs = change in credit spread.

▸ *For a fixed-rate corporate, spread duration ≈ modified duration. For a floating-rate note it is roughly the time to the next reset — which is why FRNs have near-zero rate duration but full spread duration.*

Excess return over Treasuries

$$XR \approx s \cdot t - \Delta s \cdot D_s - t \cdot p \cdot L$$

Where: s = spread, t = horizon, D_s = spread duration, p = probability of default, L = loss severity.

▸ *Carry minus the mark-to-market hit minus expected credit losses. It shows precisely how a spread widening can wipe out a full year of carry in weeks.*

Option-adjusted spread

$$OAS = Z\text{-spread} - \text{Option cost}$$

Where: Option cost from a lattice or Monte Carlo model.

▸ *OAS is the spread you compare across bonds with different embedded options — it strips out the optionality so the credit comparison is like-for-like.*

CDS-cash basis

$$\text{Basis} = \text{CDS spread} - \text{Z-spread on the cash bond}$$

Where: Both in basis points.

▸ *A negative basis (CDS cheaper than cash) lets you buy the bond AND the protection for a near-riskless spread pickup. It appeared, memorably, in 2008 when funding costs made the trade impossible to hold.*

FX-hedged USD-equivalent OAS

$$OAS_{USD\text{-}eq} = OAS_{foreign} - (r_{base} - r_{foreign})$$

Where: The differential is the rolling FX-forward hedge cost.

▸ *Comparing raw OAS across currencies is meaningless. Convert to a common currency first — otherwise you are just comparing interest-rate differentials.*

Conditional value at risk (expected shortfall)

$$CVaR_\alpha = E[L \mid L > VaR_\alpha]$$

Where: The average loss GIVEN that VaR is breached.

▸ *VaR tells you the threshold; CVaR tells you how bad it gets beyond it. For fat-tailed credit portfolios, that difference is the whole risk story.*

EXAM TIPS — CREDIT STRATEGIES

- Spread duration is the credit analogue of interest-rate duration. A portfolio can be rate-neutral and still deeply exposed to a spread widening.
- To express a credit view efficiently: cash bonds carry both rate and spread risk; CDS isolates spread risk; a CDS index gives you beta to the whole market. Choose deliberately and say why.

TRADE STRATEGY & EXECUTION · 7 formulas

Implementation shortfall, component by component. Highly examinable and very mechanical.

Implementation shortfall

$$IS = (\text{Paper portfolio gain} - \text{Actual portfolio gain}) / \text{Investment decision value}$$

Where: Components: delay + trading + opportunity cost, plus explicit fees.

▸ *The paper portfolio executes instantly and free at the decision price. Everything that separates you from it is a cost, including the shares you never got.*

Delay cost

$$\text{Delay} = (P_{arrival} - P_{decision}) \times S_{filled}$$

Where: $P_{decision}$ = PM's decision price; $P_{arrival}$ = price when the order reaches the desk.

▸ *The cost of the order sitting in someone's inbox. Invisible on a broker report, and often the largest component in a slow-moving organisation.*

Trading (market impact) cost

```
Trading = (P_exec − P_arrival) × S_filled
```

Where: Price move between arrival and execution.

▸ *This is the cost you cause. Temporary impact reverses after you stop; permanent impact does not, because you revealed information.*

Opportunity cost

```
Opportunity = (P_close − P_decision) × S_unfilled
```

Where: The return you missed on shares that never got filled.

▸ *The cost of NOT trading. A patient limit-order strategy minimises impact and maximises this — which is exactly the trade-off the exam wants you to articulate.*

Explicit fees

```
Fees = c × S_exec + taxes
```

Where: c = commission per share.

▸ *The only component that appears on an invoice, and usually the smallest. Candidates over-weight it; the exam does not.*

Market impact cost

```
Market impact = (P_exec − P_pre) / P_pre
```

Where: P_pre = pre-trade benchmark price.

▸ *Higher urgency means more impact. This is the single trade-off that determines algorithm choice.*

VWAP benchmark

$$\text{VWAP} = \Sigma(P_t \times V_t) / \Sigma V_t$$

Where: Volume-weighted average price over the period.

▸ *Easy to game (just trade with the volume) and meaningless for an order that IS a large share of the day's volume. Implementation shortfall is the more honest benchmark, and the exam prefers it.*

> **EXAM TIPS — TRADE STRATEGY & EXECUTION**
>
> - Match the algorithm to the motive. Information-driven, urgent trade ⇒ take liquidity, accept impact. Liquidity-driven, patient trade ⇒ provide liquidity, accept opportunity cost risk.
> - Implementation shortfall captures the full cost including unfilled shares; VWAP does not. When a question compares the two benchmarks, that is the answer.

CASE STUDY — ENDOWMENTS & LONG-HORIZON INVESTORS · 5 formulas

Spending rules, illiquidity budgets and hedging. The classic Level III case-study material.

Effective spending rate

$$r\_eff = S_t / MV_t$$

Where: S_t = current distribution, MV_t = current portfolio value.

▸ *In a drawdown, a fixed dollar distribution mechanically pushes the effective spending rate up — exactly when the portfolio can least afford it. That pro-cyclicality is the whole reason smoothing rules exist.*

Endowment smoothing spending rule

$$S_t = w \times S_{t-1}(1 + \pi) + (1 - w) \times r \times MV_t$$

Where: w = smoothing weight, r = target rate.

▸ *Blends last year's inflation-adjusted spend with a percentage of current market value. A high w gives the university a stable budget; a low w protects the corpus. That is the trade-off to name.*

Endowment required nominal return

```
R_req = s + π + c
```

Where: s = spending rate, π = expected inflation, c = investment costs.

▸ *Show all three terms even if one is small. Graders award marks per component, and omitting the cost term is a needless drop.*

Illiquidity budget

$$IB = TP - (Sp + CC + RB + SR)$$

Where: TP = total portfolio, Sp = spending reserve, CC = capital calls, RB = rebalancing buffer, SR = stress reserve.

▸ *What is left over is what you can afford to lock up. Endowments that skipped the stress reserve were the ones forced to sell PE stakes at a discount in 2009.*

DV01-based hedge ratio

$$N = -DV01\_portfolio / DV01\_hedge$$
$$\text{Cross-hedge:}\quad N = -(DV01\_p / DV01\_h) \times \beta\_spread$$

Where: The negative sign means short the hedging instrument.

▸ *For a cross-hedge, the beta adjustment accounts for the hedged instrument moving less than one-for-one with the position. Omitting it leaves you systematically under-hedged.*

EXAM TIPS — CASE STUDY — ENDOWMENTS & LONG-HORIZON INVESTORS

- Long-horizon investors can bear illiquidity — but only to the extent they have modelled the cash flows. Horizon is a licence to take illiquidity risk, not a substitute for liquidity planning.
- Every endowment case study is the same question in a new costume: can this portfolio meet its spending obligation, in real terms, without impairing the corpus? Answer that, then support it.

CORE 1 — CAPITAL MARKET EXPECTATIONS & ASSET ALLOCATION · 23 formulas

Forecasting inputs, then turning them into a policy portfolio. Sat by every Level III candidate regardless of pathway.

Mean-variance optimal portfolio weights

$$w^* = (1/\lambda) \times \Sigma^{-1} (\mu - r\_f \cdot 1)$$

Where: λ = risk aversion, Σ = covariance matrix, μ = expected returns.

▸ *MVO is famously input-sensitive: small changes in μ produce wild swings in weights, which is exactly why Black-Litterman and resampling exist.*

Corner portfolio blending

$$w\_A = [E(R\_P) - E(R\_B)] / [E(R\_A) - E(R\_B)]$$
$$w\_B = 1 - w\_A$$

Where: A and B = the two adjacent corner portfolios bracketing the target return.

▸ *Any blend of two adjacent corner portfolios sits on the efficient frontier. Blending non-adjacent ones does not.*

Black-Litterman expected returns

$$\text{Equilibrium:}\quad \Pi = \delta \cdot \Sigma \cdot w\_mkt$$
$$\text{Blended:}\quad E(R) = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} [(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$$

Where: δ = risk aversion, w_mkt = market-cap weights, Q = your views, Ω = confidence in them.

▸ *You will not compute the matrix algebra in the exam. You will be asked why it beats raw MVO: it anchors on market equilibrium and tilts only where you have a view.*

Rebalancing trigger (range-based)

$$\text{Rebalance when } |w_i - w^*_i| > \Delta_i$$

Where: w^*_i = target weight, Δ_i = tolerance band.

▸ *Wider bands mean lower transaction costs but more drift from policy. Bands widen with higher transaction costs and with higher correlation to the rest of the portfolio.*

Grinold-Kroner expected equity return

$$E(R) = D/P - \Delta S + g + \Delta(P/E)$$

Where: D/P = dividend yield, ΔS = net share issuance, g = nominal earnings growth, $\Delta(P/E)$ = repricing.

▸ *Note the sign on ΔS: net buybacks (negative issuance) add to return. Income + growth + repricing — that is the whole model in three words.*

Taylor rule policy rate

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - y^*)$$

Where: r^* = neutral real rate, π^* = inflation target, $(y - y^*)$ = output gap.

▸ *A rate above the Taylor-implied level suggests restrictive policy. It is a benchmark for judging central banks, not a prediction of them.*

Covered interest rate parity

$$F / S = (1 + i_d) / (1 + i_f)$$

Where: F, S quoted as domestic per foreign.

▸ *The high-yield currency always trades at a forward discount. If it did not, you could arbitrage it — which is why CIP holds tightly in practice.*

Fixed-income expected return building blocks

$$E(R) = r_f + \pi^e + TP + CP + LP$$

Where: TP = term premium, CP = credit premium, LP = liquidity premium.

▸ *Build the required return by stacking compensations for each risk you are actually bearing. Strip out the ones you are not.*

Expected fixed-income return decomposition

$$E(R_{FI}) = YTM + \text{Roll-down} + \Delta P_{yield} - L_{credit} - L_{FX}$$

Where: ΔP = price change from your yield-curve view, L = expected losses.

▸ *Roll-down is free return on an upward-sloping curve as the bond ages into a lower yield. It disappears on a flat curve.*

GARCH(1,1) variance forecast

$$\sigma^2_t = \omega + \alpha \cdot \varepsilon^2_{t-1} + \beta \cdot \sigma^2_{t-1}$$

Where: ω = long-run anchor, α = weight on the last shock, β = weight on the prior variance.

▸ *This is why volatility clusters. α + β close to 1 means shocks decay slowly and today's turbulence persists.*

Singer-Terhaar blended risk premium

$$RP = w(\sigma \cdot \rho \cdot SR_g) + (1 - w)(\sigma \cdot SR_g)$$

Where: w = degree of global market integration, ρ = correlation with the global portfolio.

▸ *A fully integrated market only earns compensation for its correlated (systematic) risk. A fully segmented one earns compensation for all its risk — hence the higher premium.*

TAA band around the strategic allocation

$$w_{TAA} \in [w_{SAA} - b, w_{SAA} + b]$$

Where: b = the tactical band set in the IPS (often ±5%).

▸ *Tactical deviations must live inside the IPS bands. A recommendation that breaches them is wrong no matter how good the market view is.*

Net after-cost tactical premium

$$\alpha_{net} = \alpha_{gross} - TC - T$$

Where: TC = transaction costs, T = realised tax cost.

▸ *Tactical calls are gross-alpha ideas that die on net-alpha arithmetic. Always net down before recommending the trade — this is a standard essay point.*

Stress liquidity coverage ratio

$$LCR_{stress} = \text{Accessible liquid assets (stress)} / \text{Cash demands (stress)}$$

Where: Both measured under a stressed scenario, not a normal one.

▸ *For illiquid-heavy portfolios, a prudent floor is around 2×. Capital calls arrive precisely when markets are down — the denominator and numerator move together, against you.*

Endowment spending rate

$$s = \text{Annual spending} / \text{Portfolio value}$$

Where: e.g. 5% of $800M = $40M.

▸ *The spending rate is the anchor for the required return. Raise spending and you must raise required return, which means raising risk.*

Correlation inputs required for MVO

$$N_\rho = n(n - 1) / 2$$

Where: n = number of asset classes.

▸ *Ten asset classes need 45 correlations. This combinatorial explosion is the practical argument against very granular MVO.*

Total MVO inputs required

$$N = 2n + n(n - 1)/2$$

Where: n expected returns + n standard deviations + n(n−1)/2 correlations.

▸ *Every extra input is an extra estimation error you feed into an optimiser that will happily maximise it.*

Pension funded ratio

$$FR = A / L$$

Where: A = market value of plan assets, L = PV of liabilities (e.g. PBO).

▸ *A well-funded plan should de-risk — it has already won. Under-funded plans face pressure to take more risk, which is exactly when they can least afford it.*

Pension surplus

$$S = A - L$$

Where: Surplus optimisation maximises the return on S, not on A.

▸ *Focusing on asset return alone ignores that the liability is also a moving, rate-sensitive position. Surplus is the real risk being managed.*

Dollar duration

$$DD = MV \times D$$

Where: MV = market value (or PV of liabilities), D = modified/effective duration.

▸ *The unit of account for LDI hedging. You match dollar duration of assets to dollar duration of liabilities — not durations, dollar durations.*

Mean-variance utility

$$U = E(R_p) - 0.5 \times \lambda \times \sigma^2_p$$

Where: λ = risk aversion (typically 1–10).

▸ *Higher λ means variance is penalised harder, so the optimal portfolio sits further left on the frontier. Set λ = 0 and you get the maximum-return corner.*

Geometric mean approximation

$$G \approx A - 0.5\sigma^2$$

Where: A = arithmetic mean, σ^2 = variance of returns.

▸ *The volatility drag. Two portfolios with the same arithmetic mean but different volatility do not compound the same — the volatile one ends up poorer.*

Roy's safety-first ratio

$$SF = [E(R_p) - R_L] / \sigma_p$$

Where: R_L = minimum acceptable (threshold) return.

▸ *Maximise SF and, under normality, you minimise the probability of falling below R_L. Set R_L = R_f and it collapses into the Sharpe ratio.*

EXAM TIPS — CORE 1 — CAPITAL MARKET EXPECTATIONS & ASSET ALLOCATION

- Asset allocation essays are graded on justification, not arithmetic. State the constraint, state the number, state the conclusion — in that order.
- Whenever a question describes an investor, extract five things before you compute anything: return requirement, risk tolerance, time horizon, liquidity needs, and legal/tax/unique constraints.
- Volatility drag ($G \approx A - 0.5\sigma^2$) is the quiet reason risk control creates return. Expect it to be tested indirectly, through a comparison of two portfolios.

CORE 2 — RISK BUDGETING, TRADING & INVESTOR TYPES · 28 formulas

Decomposing portfolio risk, measuring execution cost, and the return requirements of each institutional and private investor type.

Marginal contribution to risk

$$MCTR_i = \beta_i \times \sigma_p$$
$$\beta_i = Cov(R_i, R_p) / \sigma^2_p$$

Where: Risk added by a marginal increase in asset i's weight.

▸ *Note the beta is to the portfolio, not to the market. This is the derivative of portfolio risk with respect to weight.*

Absolute contribution to risk

$$ACTR_i = w_i \times MCTR_i$$
$$\Sigma\ ACTR_i = \sigma_p$$

Where: The individual contributions sum exactly to total portfolio risk.

▸ *Risk budgeting means setting target ACTRs. An asset with a small weight can dominate the risk budget if its volatility and correlation are high.*

Tracking error (active risk)

$$TE = \sigma(R_p - R\_B)$$
$$TE\_annual = TE\_monthly \times \sqrt{12}$$

Where: Standard deviation of the active return series.

▸ *Scale by √12 from monthly, √252 from daily. Scaling the mean uses 12; scaling the standard deviation uses √12 — do not mix them.*

Implementation shortfall

$$IS = Explicit + Delay + Impact + Opportunity$$

Where: Explicit = commissions and fees; delay = decision-to-desk drift; impact = the price move you cause; opportunity = the return on shares you never filled.

▸ *This is the complete cost of a trade, including the cost of not trading. Trading urgently kills you on impact; trading slowly kills you on opportunity cost.*

Effective spread

$$Effective\ spread = 2 \times |P\_exec - P\_mid|$$

Where: P_mid = quote midpoint at the time of execution.

▸ *The factor of 2 makes it round-trip comparable to the quoted spread. Effective spread below quoted spread means you got price improvement.*

Square-root market impact

$$Impact \propto \sqrt{shares / ADV}$$

Where: ADV = average daily volume.

▸ *Doubling order size raises impact by about 41%, not 100%. This concavity is why breaking large orders into pieces helps — but only up to a point.*

VWAP transaction cost

$$\text{Buy:}\quad cost = P\_exec - VWAP$$
$$\text{Sell:}\quad cost = VWAP - P\_exec$$

Where: Positive means unfavourable in both cases.

▸ *VWAP flatters a patient trader and is easy to game. It is meaningless for an order that is itself a large share of the day's volume.*

Effective equity beta including private equity

$$\beta_{eff} = w_{eq} + w_{PE} \times \beta_{PE}$$

Where: $\beta_{PE} \approx 1.3$ (PE has levered equity beta).

▸ *Reported PE returns are smoothed by appraisal-based NAVs, which understates true beta. A portfolio that looks defensive may be levered equity in a slow-reporting wrapper.*

Effective PE allocation including unfunded commitments

$$w_{PE}(eff) = (NAV + UC) / (P + UC)$$

Where: UC = unfunded commitments, P = total portfolio value.

▸ *Ignoring unfunded commitments systematically understates PE exposure. The capital is committed — you just have not written the cheque yet.*

Total PE economic exposure

$$E_{PE} = NAV + UC$$

Where: What you are really on the hook for.

▸ *The denominator changes but the message does not: commitments are exposure. This is what caught endowments out in 2008.*

Liquidity coverage for an alternatives programme

$$LCR = L / CF_{12m}$$

Where: L = liquid assets, CF_{12m} = committed cash outflows over the next 12 months (capital calls + benefit payments).

▸ *The denominator includes both. An endowment paying a spending distribution and facing capital calls in the same drawdown is the classic liquidity failure.*

Endowment nominal return requirement

$$R_{nom} = s + c + \pi$$

Where: s = spending rate, c = management costs, π = expected inflation.

▸ *Additive approximation. Purists multiply: $(1 + s)(1 + c)(1 + \pi) - 1$. The exam accepts the additive form unless it says otherwise.*

Endowment real return requirement

$$R_{real} = s + c$$

Where: Inflation stripped out.

▸ *This is what "preserving intergenerational equity" actually requires: earn the spend plus the costs, in real terms, forever.*

Foundation minimum return

$$R \geq 0.05 + c + \pi$$

Where: 5% = the US IRS minimum annual distribution requirement.

▸ *The 5% is a legal floor, not a policy choice. It is the key structural difference between a private foundation and an endowment.*

Insurer duration-matched immunisation

$$D_A \times A = D_L \times L$$

Where: Match dollar duration of assets to dollar duration of liabilities.

▸ *Duration matching only immunises against small, parallel shifts. Convexity matching and periodic rebalancing are what make it hold.*

DV01 (price value of a basis point)

$$DV01 = D_{mod} \times MV \times 0.0001$$

Where: Dollar change in value per 1 bp yield move.

▸ *The hedging currency of LDI. Number of futures = ΔDV01 required ÷ DV01 per contract.*

Active share

$$AS = \tfrac{1}{2} \Sigma |w_{p,i} - w_{b,i}|$$

Where: Summed across the combined portfolio-plus-benchmark universe.

▸ *Active share and tracking error are different things. A well-diversified stock-picker can have high active share and low tracking error; a sector bet has the reverse.*

Required pre-tax return for a private client

$$r = [(S - I)/V + \pi] / (1 - t) + f$$

Where: S = spending, I = other income, V = portfolio value, t = tax rate, f = advisory fees.

▸ *Order of operations is the whole question: net the spending against income, express as a rate, add inflation, gross up for tax, then add fees. Grossing up in the wrong place is the standard error.*

Leveraged portfolio return on equity

$$r_p = r_i + (V\_B / V\_E)(r_i - r\_B)$$

Where: V_B = borrowed value, V_E = equity, r_B = borrowing cost.

▸ *Leverage multiplies the spread between asset return and borrowing cost — in both directions. If r_i < r_B, leverage accelerates the loss.*

Human capital

$$HC = \Sigma\ E[w_t] / (1 + r)^t$$

Where: w_t = expected labour income, r = risk-adjusted discount rate.

▸ *A tenured professor has bond-like human capital and can hold more equities. A commission-based trader has equity-like human capital and should hold fewer. This is the entire life-cycle argument.*

Real after-tax return

$$r\_real,at \approx r\_nom - \pi - t \times r\_nom$$

Where: Tax is levied on the nominal gain, including the inflation component.

▸ *Tax on inflation is a real loss of purchasing power. In high-inflation regimes a positive nominal after-tax return can still be a real loss.*

Modified duration from Macaulay

$$D\_mod = D\_Mac / (1 + y)$$

Where: y = periodic yield to maturity.

▸ *Carried straight over from Level I and still tested. Effective duration replaces it wherever cash flows are contingent.*

Taxable-equivalent yield

$$TEY = y\_muni / (1 - t)$$

Where: t = investor's marginal tax rate.

▸ *The higher the client's bracket, the more attractive the muni. Compare TEY, never the raw muni yield, to a taxable bond.*

Economic net worth

$$ENW = HC + FC - PV(Liabilities)$$

Where: The holistic balance sheet.

▸ *Institutional thinking applied to an individual: assets include the PV of future earnings, liabilities include the PV of future consumption.*

SWF stabilisation fund sizing

$$AUM\_stab = f \times G \times n$$

Where: f = fiscal dependence on the commodity, G = annual government spending, n = years of shortfall to cover.

▸ *A stabilisation fund is a short-horizon, low-volatility mandate — it may be called on next year. Do not let it hold illiquid growth assets.*

Norway-style fiscal transfer rule

$$T\_annual = r\_real \times V\_fund$$

Where: r_real ≈ 3% expected real return.

▸ *Spending only the expected real return preserves the principal in perpetuity. It is a savings fund, not a stabilisation fund — different horizon, different portfolio.*

Commodity SWF sector exposure cap

$$w\_energy \le w\_bench - \Delta$$

Where: Δ = a deliberate underweight (e.g. 30% of the benchmark weight).

▸ *The fund's inflows are already correlated with oil. Holding benchmark energy weight doubles down on the risk the country already carries — this is the concentration argument.*

SWF withdrawal cap

$$W_{max} = c \times (\text{3-year average AUM})$$

Where: c = charter cap (often 5%).

▸ *Averaging AUM over three years smooths withdrawals so a single bad market year does not force a spending collapse — the same logic as endowment smoothing.*

EXAM TIPS — CORE 2 — RISK BUDGETING, TRADING & INVESTOR TYPES

- Trading questions almost always resolve to the urgency trade-off: high urgency ⇒ market impact dominates; low urgency ⇒ opportunity cost and delay dominate. Match the algorithm to the motive for the trade.
- For any investor type, the exam wants: time horizon → liquidity need → risk tolerance → required return, in that causal order. Do not lead with the return number.
- Unfunded commitments are exposure. Any question featuring an endowment with a large PE programme is testing whether you know that.

CORE 3 — PERFORMANCE MEASUREMENT & ATTRIBUTION · 12 formulas

Risk-adjusted ratios and the Brinson decomposition. Reliably examined, and easy marks if you know which denominator to use.

Information ratio

$$IR = (\bar{R}_p - \bar{R}_B) / TE = \bar{\alpha} / TE$$

Where: $\bar{\alpha}$ = mean active return, TE = tracking error.

▸ *Active return per unit of active risk. Unlike Sharpe, IR is unaffected by adding cash or leverage to an unconstrained portfolio.*

Fundamental law of active management

$$IR = IC \times \sqrt{BR}$$
$$E(R_A) = IC \times \sqrt{BR} \times \sigma_A$$

Where: IC = information coefficient (skill), BR = breadth (independent bets per year).

▸ *Skill matters more than breadth because breadth is under a square root. Note that breadth must be independent — 500 correlated bets are not 500 bets.*

Sharpe ratio

$$SR_p = (R_p - R_f) / \sigma_p$$
$$SR^2_{combined} = SR^2_B + IR^2$$

Where: Total risk in the denominator.

▸ *The second identity is the beautiful one: adding an active overlay to a benchmark improves the combined Sharpe by exactly the manager's IR, squared.*

M-squared

$$M^2 = (R_p - R_f) \times (\sigma_m / \sigma_p) + R_f$$

Where: De-levers or levers the portfolio to the market's volatility.

▸ *Same ranking as Sharpe, but expressed in percent — so it can be compared directly against the market return.*

Treynor ratio

$$T = (R_p - R_f) / \beta_p$$

Where: Systematic risk in the denominator.

▸ *Correct when the portfolio is one sleeve of a diversified whole. Sharpe is correct when it is the client's entire wealth.*

Sortino ratio

$$Sortino = (R_p - MAR) / \sigma_d$$

Where: MAR = minimum acceptable return, σ_d = downside deviation below MAR.

▸ *Only penalises downside volatility. Preferred for strategies with deliberately asymmetric payoffs — option-selling, hedge funds, anything with a fat left tail.*

Brinson allocation effect

$$A_i = (w_{p,i} - w_{b,i})(R_{b,i} - R_b)$$

Where: Was the sector overweighted, and did that sector beat the total benchmark?

▸ *Allocation uses BENCHMARK sector returns. Being overweight a sector that outperformed the overall benchmark is a positive allocation effect — regardless of what you actually picked inside it.*

Brinson selection effect

$$S_i = w_{b,i} \times (R_{p,i} - R_{b,i})$$

Where: Did your picks beat the sector benchmark?

▸ *Selection uses BENCHMARK weights. Allocation ⇒ weight difference × benchmark return. Selection ⇒ benchmark weight × return difference. Getting these backwards is the most common attribution error there is.*

Upside capture ratio

$$UC = \bar{R}_{p,up} / \bar{R}_{b,up}$$

Where: Averaged over periods when the benchmark rose.

▸ *Above 100% means the manager amplifies up markets.*

Downside capture ratio

$$DC = \bar{R}_{p,down} / \bar{R}_{b,down}$$

Where: Averaged over periods when the benchmark fell.

▸ *Below 100% means the manager dampens losses — which is the more valuable of the two, because of compounding.*

Up/down capture ratio

$$\text{Up/Down} = \text{Upside capture} / \text{Downside capture}$$

Where: A single asymmetry number.

▸ *Above 1.0 is favourable convexity. A manager capturing 90% of the upside and 60% of the downside (ratio 1.5) beats one capturing 110% and 105%.*

Symmetric performance fee

$$\text{Fee} = \text{Base} + s \times (R_p - R_B)$$

Where: s = sharing rate.

▸ *Symmetric means the fee falls when the manager underperforms — genuine alignment. Asymmetric (upside-only) fees give the manager a free option on volatility, and the exam wants you to say so.*

> **EXAM TIPS — CORE 3 — PERFORMANCE MEASUREMENT & ATTRIBUTION**
>
> • Attribution: allocation is about WEIGHTS, selection is about PICKS. If you remember only one thing from this section, remember which return goes with which weight.
>
> • Choosing the ratio is the question. Total risk ⇒ Sharpe or M^2. Systematic risk ⇒ Treynor or Jensen. Active risk ⇒ information ratio. Downside risk ⇒ Sortino.
>
> • A high information ratio from a very low tracking error is fragile — it may just be a closet indexer with a lucky quarter. Look at active share alongside it.

CORE 4 — DERIVATIVES & CURRENCY RISK MANAGEMENT · 15 formulas

Hedging, overlays and option structures. The futures-contract formulas are near-guaranteed calculation marks.

Option delta

$$\text{Call: } \Delta_c = N(d_1) \in (0, 1)$$
$$\text{Put: } \Delta_p = N(d_1) - 1 \in (-1, 0)$$
$$\Delta_c - \Delta_p = 1$$

Where: Approximate change in option value per $1 move in the underlying.

▸ *Delta is also a rough probability of finishing in the money. ATM options sit near |0.5| and their delta moves fastest — that is gamma.*

Protective put

$$\text{Payoff} = S_T + \max(X - S_T, 0) = \max(S_T, X)$$
$$\text{Profit} = \text{Payoff} - (S_0 + p)$$

Where: Long stock + long put.

▸ *Insurance: floor the downside, keep the upside, pay a premium for the privilege. The put premium is the drag on a flat market.*

Covered call

$$\text{Payoff} = S_T - \max(S_T - X, 0) = \min(S_T, X)$$
$$\text{Profit} = \text{Payoff} - S_0 + c$$

Where: Long stock + short call.

▸ *Income in flat or mildly down markets, capped upside. You are selling the right tail to fund the present — fine until the stock gaps up.*

Collar

$$\text{Payoff} = S_T + \max(X_L - S_T, 0) - \max(S_T - X_H, 0)$$
$$= \min(\max(S_T, X_L), X_H)$$

Where: Long stock + long put (X_L) + short call (X_H).

▸ *A zero-cost collar funds the protective put by selling the call. Standard answer for a concentrated stock position the client cannot or will not sell.*

Put-call parity

$$S + P = C + PV(K)$$

Where: Same strike, same expiry, European.

▸ *Every synthetic in the Level III toolkit falls out of rearranging this. Long stock + long put = long call + cash.*

Long straddle breakevens

$$BE = K \pm (C_0 + P_0)$$

Where: Long call + long put at the same strike.

▸ *A pure volatility bet — you need a big move in either direction. You lose if the underlying sits still, and time decay is working against you the whole way.*

Bull call spread maximum gain

$$\text{Max gain} = (K_H - K_L) - \text{Net premium}$$

Where: Buy the lower strike, sell the higher.

▸ *Max loss is simply the net premium paid. Capping the upside is what makes the position cheap.*

Maximum loss on short stock + long call

$$\text{Max loss} = K - S_0 + C_0$$

Where: A synthetic long put.

▸ *The call caps the unlimited loss on the short. Recognising the synthetic is usually faster than working the payoff from scratch.*

Number of bond futures to adjust duration

$$N = (DD_T - DD_P) / DD_f$$

Where: DD = dollar duration; DD_f is BPV-adjusted by the conversion factor.

▸ *Positive N means buy futures to extend duration; negative means sell to shorten. The conversion factor for the cheapest-to-deliver bond is not optional.*

Number of equity futures to adjust beta

$$N = [(\beta_T - \beta_P) / \beta_F] \times [V / (P_f \times m)]$$

Where: β_T = target beta, m = contract multiplier.

▸ *Set $\beta_T = 0$ to fully hedge equity exposure; set $\beta_T = 1$ to equitise a cash pile. Same formula, both directions.*

Roll yield on a currency forward hedge

$$\text{Roll yield} = (F - S) / S$$

Where: Approximately the domestic minus foreign interest rate differential.

▸ *Hedging a high-yielding foreign currency costs you money (negative roll). This is why the hedging decision is never free, and why the exam asks about it.*

Minimum-variance hedge ratio

$$h^* = \rho_{A,B} \times (\sigma_A / \sigma_B)$$

Where: A = the asset being hedged, B = the hedging instrument.

▸ *It is just the slope coefficient from regressing A on B. If $\rho = 1$ and volatilities match, $h^* = 1$ and it is a perfect hedge — real cross-hedges never are.*

Domestic-currency return on a foreign asset

$$R_{DC} = (1 + R_{FC})(1 + R_{FX}) - 1$$

Where: R_{FX} = % change in the foreign currency against the domestic.

▸ *It is multiplicative, not additive. The cross term matters when both moves are large — and the exam picks large numbers deliberately.*

Variance notional from vega notional

$$N_{var} = N_{vega} / (2 \times \sigma_{strike})$$

Where: σ_{strike} in whole-number percent (e.g. 20, not 0.20).

▸ *Vega notional is intuitive (dollars per vol point); variance notional is what the contract actually pays on. Convert before you compute the payoff.*

Variance swap payoff

$$\text{Payoff} = N_{var} \times (\sigma^2_{realised} - \sigma^2_{strike})$$

Where: Volatilities squared — it settles on variance, not volatility.

▸ *Because it pays on variance, the payoff is convex in volatility: the long side gains more from a vol spike than it loses from an equivalent vol fall.*

EXAM TIPS — CORE 4 — DERIVATIVES & CURRENCY RISK MANAGEMENT

- Futures overlay questions are formulaic marks. Write down current exposure, target exposure, and the exposure per contract — then divide. The only real difficulty is a sign error.
- Currency hedging: the roll yield tells you the cost, the correlation tells you the benefit. A currency that already hedges the underlying asset (negative correlation) may be best left unhedged.
- When a question describes a concentrated low-basis stock position, it wants a collar, an exchange fund, or a completion portfolio — and it wants you to discuss the tax consequence of each.

LEVEL III EXAM STRATEGY

The constructed-response session

- Answer in the box, in bullets, using the command word. "Justify" needs a reason. "Calculate" needs the number and the working. "Determine" needs a decision first, then support.
- Allocate time by the minutes printed on the question, not by how interesting it is. Overrunning one question steals marks from an easier one you never reach.
- Show the formula, then the substitution, then the answer. If the arithmetic goes wrong, a visible method still earns most of the marks.
- Do not write an essay. Graders are looking for specific keywords, and prose buries them.

Concepts that carry the most weight

- Every IPS question: return requirement, risk tolerance, and then the five constraints (time, liquidity, tax, legal, unique).
- Duration and DV01 hedging — the calculation marks are reliable and the method is short.
- Behavioural biases: name the bias, give the evidence FROM THE VIGNETTE, and state the consequence. All three, every time.
- The GIPS provisions and the Code and Standards — high weight, and the least forgiving of vagueness.

A note on the pathways

- The core (Topics 1–4 in this sheet) is common to all three pathways. Roughly two-thirds of the exam sits there.
- Do not over-invest in the pathway at the expense of the core. Candidates fail on asset allocation and performance, not on their specialism.